PUTNAM TRAINING PROBLEMS 2001.4
Oh yeah? Well generalise this!

1. Evaluate the determinant for the $n \times n$ matrix whose i,j-th entry is a_i^{j-1}, for predetermined values $a_1, \ldots a_n$.

2. Given that $\int_0^\infty (\sin x)/x \, dx = \pi/2$, evaluate $\int_0^\infty (\sin^2 x)/(x^2) \, dx$.

3. (Putnam 1982.) Evaluate
$$\int_0^\infty \frac{\arctan(\pi x) - \arctan x}{x} \, dx.$$

4. Which is larger: $\sqrt[3]{60}$ or $2 + \sqrt[3]{7}$?

5. Given a set of 51 integers between 1 and 100 inclusive, show that at least one member of the set must divide another member of the set.

6. Sum the series
$$\sum_{n=0}^{\infty} \frac{(n+1)^2}{n!}.$$

7. On $[0,1]$, let f be a real-valued function with continuous derivative satisfying $0 < f'(x) < 1$. Suppose that $f(0) = 0$. Prove that
$$\left[\int_0^\infty f(t) \, dt\right]^2 \geq \int_0^1 [f(t)]^3 \, dt.$$

8. Show that some multiple of the integer 17623176 involves all ten digits.

9. Let $\pi(n)$ be the number of primes not greater than n. Prove that $\pi(n)$ divides n for infinitely many n.

10. (Putnam 1990.) Is $\sqrt{2}$ the limit of a sequence of numbers of the form $n^{1/3} - m^{1/3}$ $(n, m = 0, 1, 2, \ldots)$? Justify your answer.