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Measurement and Transfer of Catastrophic Risks. A Simulation Analysis

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Abstract

When analyzing catastrophic risk, traditional measures for evaluating risk, such as the probable maximum loss (PML), value at risk (VaR), tail-VaR , and others, can become practically impossible to obtain analytically in certain types of insurance, such as earthquake, and certain types of reinsurance arrangements, specially non-proportional with reinstatements. Given the available information, it can be very difficult for an insurer to measure its risk exposure. The transfer of risk in this type of insurance is usually done through reinsurance schemes combining diverse types of contracts that can greatly reduce the extreme tail of the cedant's loss distribution. This effect can be assessed mathematically. The PML is defined in terms of a very extreme quantile. Also, under standard operating conditions, insurers use several "layers" of non proportional reinsurance that may or may not be combined with some type of proportional reinsurance. The resulting reinsurance structures will then be very complicated to analyze and to evaluate their mitigation or transfer effects analytically, so it may be necessary to use alternative approaches, such as Monte Carlo simulation methods. This is what we do in this paper in order to measure the effect of a complex reinsurance treaty on the risk profile of an insurance company. We compute the pure risk premium, PML as well as a host of results: impact on the insured portfolio, risk transfer effect of reinsurance programs, proportion of times reinsurance is exhausted, percentage of years it was necessary to use the contractual reinstatements, etc. Since the estimators of quantiles are known to be biased, we explore the alternative of using an Extreme Value approach to complement the analysis.

KEY WORDS: Quantile, Extreme Value, Monte Carlo Methods, PML, VAR, Reinsurance.

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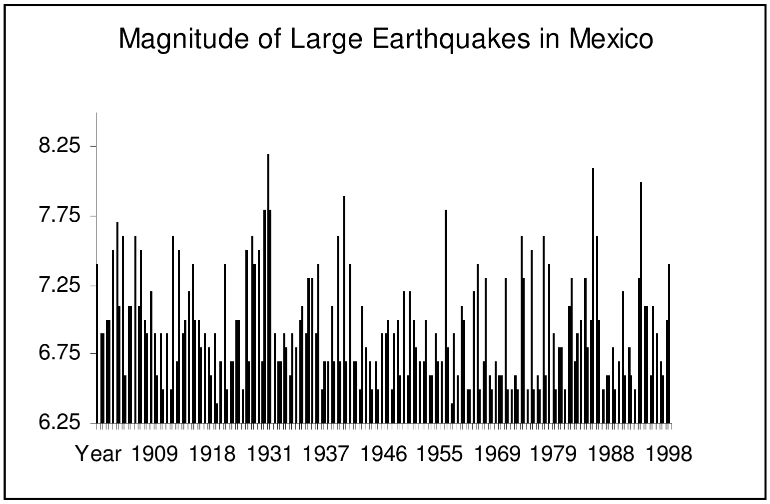
Introduction

The measurement and transfer of risk are at the essence of the insurance business. This has prompted the development of quantitative techniques to achieve both. They are important for all the stakeholders in the industry: the direct insurer, a potential reinsurer, regulators, rating agencies, and consumers. In the case of catastrophic risks (defined for the purpose of this paper as those with low frequency and high severity), they become particularly relevant due to the magnitude of potential losses, Woo (1999). A large earthquake or hurricane (or sequence of them) will impact losses in an extreme fashion, such that if not adequately reserved and capitalized, or covered by reinsurance or retrocession, it can cause the ruin of either the insurer or the reinsurer, with 'catastrophic' consequences for stockholders and society. Hence the importance of measuring this kind of risk and its transfer.

Mexico is a country with a large number of earthquakes per year. On average, there are 80 of magnitude larger than 4.3 every year. The available information and models allow us to analyze the information on earthquake intensities over the last 100 years. The National Seismological Service of Mexico, SSN (1999), has published the magnitudes of earthquakes larger than 6.5 on the Richter scale during the 20th century, see Figure 1.

Figure 1



It is clear that the correct evaluation of the potential catastrophic losses due to large earthquakes is of great importance. There have been large earthquakes in 1932 (8.2), 1942 (7.9), 1957 (7.8), 1985 (8.1) and 1995 (8.0). The largest losses were due to the 1985 earthquake, as a result of the location of its epicenter off Acapulco, more than 300 kilometers away from Mexico City, but with catastrophic local effects in Mexico City. By some accounts it was the strongest earthquake to hit Mexico in the twentieth century. It is clearly of great importance to have adequate models to evaluate and measure this kind of risk. In addition, Mexico is one of the few countries that is also subject to large hurricane losses, in the Caribbean and to a lesser extent, in the Pacific Ocean. As recently as 2005, there were huge losses due to the impact of hurricanes Emily and Wilma, mostly in the Yucatán peninsula. However, in this paper we focus on earthquake risk.

Catastrophic Risk Measurement

In many countries earthquake catastrophic risk is measured in terms on the probable maximum loss (PML), an extreme quantile of the corresponding loss distribution. Quantile-based risk measures such as PML (another name for a specific VaR) and tail-VaR have been in use for a long time now to measure risk in insurance contexts, Dowd and Blake (2006), and the PML is the ‘official’ measure for catastrophic risk in Mexico. Given the available information it can be very difficult for an insurer to measure this risk. However, since the distribution of losses due to earthquakes for a large portfolio of risks will usually be unknown the only way to quantify risk may be through simulation. That is the process we follow here.

In insurance terminology, the PML is defined as the losses y_0 that will be observed with a probability less than or equal to some previously agreed value, say *0.002. = 1/500*, so that *Pr{losses≥PML} = 1/500*, the exceedance probability. However, the PML is usually expressed in terms of a return period, the time between events of a given magnitude, and this is defined as the inverse of the exceedance probability, i.e. Return period = *1/Pr{losses ≥ PML} = 500*. Since these probabilities are usually very small they are more difficult to compute.

The methodology for estimating probable maximum loss (PML) for natural catastrophes has slowly evolved over the past few decades from being deterministic to one based on stochastic models. For a review of the development process see Woo (2002). In all cases, however, a guiding principle has been that any technique for computing PML should reflect the dual hazard and vulnerability aspects of loss. An essential flaw in deterministic models is that they exclude numerous earthquake sources that collectively contribute to the earthquake risk to the portfolio. Not just one, but a considerable number of sites may each produce earthquakes capable of causing major losses to a given portfolio. A multiplicity of hazard sources should be taken into account. The approach described in this paper is a wholly probabilistic one.

One reason for the slow evolution is certainly the fact that the processes of earthquake generation, shock wave diffusion, damage to buildings, etc. are very complex and their interaction makes their analysis even more so. It is necessary to bring together the expertise of geophysicists, structure engineers, actuaries, financial experts and others in order to construct a model that represents the overall process reasonably well. It is interesting to note that, as pointed out by Woo (2002) " *The rate of progress was slowed by the reluctance of regulators…*".

Clark (2002) explains how computer models can be used in estimating catastrophe losses. She points out the various components of such models and stages in their construction and application. Those components are present in the modeling process followed by ERN to develop the models used here. However, in addition to the problems inherent in the modeling process and estimation we must also face the fact that the usual risk measures are not coherent McNeil et al. (2005). In particular this is true of the PML and VaR even though they are extensively utilized. It is also known that quantile based measures are biased, Inui et al. (2005) and Kim and Hardy (2007). We will complement the simulation analysis with some coherent risk measures: the Conditional Tail Expectation or tail-VaR, Artzner et al. (1999).

McNeil et al. (2005) distinguish several approaches to measuring risk. Here we concentrate on risk measures based on the loss distribution since this is how earthquake catastrophic risk is generally measured. They indicate that one of the main problems in working with distributions is that it is difficult to estimate the loss distribution. In particular, when analyzing catastrophic risk, traditional measures for evaluating risk, such as the probability of probable maximum loss (PML), value at risk (VaR) (both are quantiles of the distribution), tail-VaR (also known as Conditional Tail Expectation, or CTE) and others can become nearly impossible to obtain analytically in certain types of insurance, such as earthquake where normally losses are due to a main event and a replica. Another element that complicates the analysis, and which is of key importance for the industry, is that a measure of the risk is required for a whole year of coverage, and specially with multiple areas of exposure.

Risk Transfer

Because of the magnitude of potential losses, risk mitigation in this type of insurance is usually done through diverse types of traditional reinsurance (proportional and non-proportional) and alternative (cat-bonds and the like) risk transfer schemes. To clearly understand the exposure of the portfolio and achieve effective risk mitigation from financial and regulatory points of view, proper measurements of the magnitude of the losses, with and without the risk transfer chosen, are needed.

To comply with the requirements of regulatory and accounting frameworks, such as Solvency II (which states the valuation of recoverables from reinsurance contracts and special purpose vehicles on the same basis as the valuation of the contractual obligations, and both capturing the risk profile of the portfolio), it is necessary not only to have the right measurement, but to have it available for auditing, adequately documented, and with the enough and clear elements for communication with and disclosure to interested parties (company management and board, reinsurers, authorities and rating agencies) As we will see, this approach is specially useful on the communication side.

In terms of risk management, the correct measurement of the risk of a succession of catastrophic events and not only a single one is a must, specially in countries exposed to both seismological and hydrometeorological dangers, such as the USA, Japan, México and others.

In terms of risk transfer or mitigation, it is well known that in proportional reinsurance (quota share), the insurer takes a proportion of every loss, so that if X_i is the random variable that represents gross losses then the losses net of the reinsurance is $Y = \alpha X$ where α is the retention rate. Alternatively non-proportional reinsurance (e.g. excess-loss) states that for every loss exceeding a specified threshold or priority (P), the reinsurer will pay the loss up to a certain limit (L), so that for each gross loss occurrence the direct insurer will pay only Max{0, min (X-P, L)}, Booth et al. (1999). This has the effect of truncating the loss distribution. Usually, excess-loss treaties include provisions for coverage reinstatement, after the initial coverage has been used up, in one or more events.

Non-proportional reinsurance can greatly reduce the extreme tail of the cedant's loss distribution. This effect can be assessed mathematically. If the PML is being defined in terms of a very extreme quantile we argue that in simple cases, and if there is a limit to the non-proportional reinsurance, the reduction in the PML can be very unstable, depending on the relation between the limit and the PML. Also, under standard operating conditions, insurers use several "layers" of non proportional reinsurance that will be combined with some type of proportional reinsurance. The resulting reinsurance structures will then be very complicated to analyze, Verlaak and Beirlant (2003). This is further complicated if the probability distribution of losses is not known analytically. In fact most of the literature on optimal reinsurance assumes it is known. Recently, Silvestrov et al. (2006) developed criteria for evaluating alternative reinsurance contracts that are large and mathematically complex. They use a Monte Carlo based approach.

Regulatory Considerations

It has been argued that it is impossible to measure the mitigation effect, or transfer of risk, of non proportional reinsurance and so it should not be given recognition for solvency assessment. Several reasons have been put forward for this, among others the following:

a) The difficulty of estimating their effect
b) The inclusion of aggregate limits
c) Given the fact that we are concerned not only with exposure to one event, but with exposure to a series of them, it will be absolutely necessary to have "reinstatement" clauses and a proper measure of their adequacy

Nevertheless, proper recognition of reinsurance is necessary in order to assess the risk reduction for the ceding company. This has implications for capital requirements to ensure effective solvency supervision. The Insurer Solvency Assessment Working Party (ISAWP) of the International Actuarial Association (IAA), ISAWP (2004), states that

"*While proper treatments and recognition of reinsurance arrangements are necessary to assess the impact of the of a ceding company's risk profile, this is a difficult task for a number of reasons.*

The first complexity comes from the tremendous diversity in the types of reinsurance contracts:

- *Typical reinsurance arrangement comprise both proportional and non-proportional covers*
- *Some contracts have variable rating terms, ... for a proportional reinsurance treaty, and reinstatements or contingent commissions for an excess-of-loss treaty*
- *Some contracts cover just one line of business, others cover multiple lines of business ...*
- *Some contracts are on an aggregate basis, with aggregate deductibles and aggregate limits*

- *Some financial type reinsurance contracts cover a hybrid of underwriting and financial risks.*

The second complexity comes from the fact that many reinsurance contracts do not bear a linear relationship with the underlying risks."

The ISAWG further indicates that "*the proper evaluation of the risk reducing impact of non-proportional reinsurance contracts is still not possible without either relatively complex mathematical transformations, which are typically beyond the of supervisory control mechanisms, or the use of simulations, which are standard routines for more complex risk modelling in internal models."*

In addition the ISAWP also indicates that if applied properly to evaluate the solvency of a direct insurer, reinsurance is a very efficient means of reducing risk (particularly if measured by tail-VaR) and hence can be a useful alternative for capital.

Hence when reinsurance schemes are very sophisticated, it becomes very complicated, if not impossible, to evaluate their mitigation or transfer effects analytically then it may be necessary to use alternative approaches, such as Monte Carlo simulation methods, Silvestrov et al. (2006). That is what we do in this paper in order to measure the effect of a complex reinsurance treaty on the risk profile of an insurance company.

Something that also should be taken into account is that simulations generally produce results that help make better management decisions, improving communication to different stakeholders, such as underwriters, reinsurers, the company's board and rating agencies.

The Model

In general, models developed to estimate catastrophic losses are based on the physical laws of nature that govern the specific phenomena, in our case earthquake occurrence, and on the equations that embody them. Thus by combining mathematical representations of the natural occurrence patterns and characteristics of earthquakes, with complementary information on property values, construction types, and other characteristics, as well as information on insurance and reinsurance contracts, these models can provide extensive information to companies concerning the potential for large losses before they actually occur.

In Mexico, the insurance regulatory body (Comision Nacional de Seguros y Fianzas, CNSF) has commissioned the construction of an earthquake loss model that must be used to compute the pure risk premium as well as the PML[1]. These

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[1] ERN Ingenieros Consultores, S.C. (2002), *Evaluación de Riesgo Sísmico*, Manual de Referencia.

results are used to verify compliance with corresponding regulation and compute statutory reserves.

Even better, the software produces additional output that can be used for simulation. These simulation exercises can provide a rich output that can be used for many different applications. Probability distributions of losses and their complement, exceedance probabilities, can be estimated for potential levels of annual aggregate and per-occurrence losses that a company may experience given its portfolio of property exposures, Clark (2002). There are several commercial simulation models (AIR, EQECAT, RMS) that do this. Here we intend to show how a similar model can be used by the insurance companies.

Hence, based on the arguments put forth by international associations, such as the International Association of Insurer Supervisors (IAIS) and the International Actuarial Association (IAA), the latter through the ISAWP, that encourage the use of mathematical models and simulation methods, we have used this output and constructed a program that allows the actuary to generate the distributions of gross yearly losses for an insurance portfolio. The algorithm includes the possibility of simulating

a) the occurrence of one or several earthquakes in a year
b) their impact on the insured portfolio
c) the risk transfer effect of reinsurance programs that mix different types or reinsurance
d) descriptive statistics for, gross losses, and losses net of reinsurance
e) the proportion of times the reinsurance is exhausted
f) average cost per year of reinstatements
g) distribution of loss by reinsurance layer according to their magnitude
h) percentage of years it was necessary to contract additional reinstatements

The model consists of a series of sub-models corresponding to different aspects of the earthquake loss generation process. The initial component is earthquake occurrence. This is modeled as a spatial Poisson distribution for each of a number of potential seismic sites, i.e. space has been discretized in 3600 points. Then there is the distribution of earthquake magnitudes at each one of the sites. The exceedance rate for the *i-th* site is specified as:

$$\nu_i(y) = \int_{M_0}^{M_{ui}} -\frac{d\lambda_i(M)}{dM} P(Y > y \mid M, f_i)dM = -\int_{M_0}^{M_{ui}} P(Y > y \mid M, f_i)d\lambda_i(M) \qquad (1)$$

where $\lambda_i(M)$ is the number of earthquakes of magnitude greater than M at source i. Here, $\nu_i(y)$ is the average number of events, by unit time, that produce losses larger than y at seismic source i, say f_i. Then the total exceedance rate (The average number of events that produce losses will exceed a given value y) for the whole portfolio is:

$$\nu(y) = \sum_{i=1}^{Nf} \int_{Mo}^{Mui} -\frac{d\lambda_i(M)}{dM} \Pr(Y > y \mid M, f_i) \Pr(f_i) dM \qquad (2)$$

where M_{oi} and M_{ui} are the lower and upper bounds on the magnitudes at site *i*, respectively; and N_{f} is the number of sites. So that if $\nu(0) =$ average number of events by unit time, that produce losses greater than *0*, then the probability distribution for the losses of the whole portfolio is

$$F(y) = 1 - \frac{\nu(y)}{\nu(0)}. \qquad (3)$$

In equation (2) $\Pr(Y > y \mid M, f_i)$ is obtained in the program as follows: given an earthquake of magnitude *M*, at site *i* is

$$\Pr(Y > y \mid M, f_i) = \Pr(S_{\text{exp}}\beta > y \mid M, f_i) = \Pr(\beta > y / S_{\text{exp}} \mid M, f_i)$$

where *Y* are the losses and β = relative loss, as a proportion of the total amount exposed S_{exp}, i.e. $\beta = \frac{Y}{S_{\text{exp}}}$. There is one such distribution for each 'site-magnitude' combination. These distributions are produced by the software, ERN (2002), by aggregating the corresponding distributions for β at each insured building. The corresponding densities are specified as:

$$f_B(\beta) = P_0\delta(\beta) + (1 - P_0 - P_1)B(\beta; a, b \mid M, f_i) + P_1\delta(\beta - 1) \qquad 0 \le \beta \le 1 \quad . \qquad (4)$$

In equation (4) P_0 is the probability of zero losses, P_1 the probability of total losses, *Beta*($y;a,b|M, f_i$) refers to a beta density with parameters *a,b*, that are conditional on the 'site-magnitude' combination, and δ is the Dirac delta. These distributions are obtained using information on the construction characteristics for each insured building combined with shock wave diffusion and local effects from earthquakes at the given 'site-magnitude'; they lead to a ratio damage distribution for each building. The individual distributions for the loss proportions in each building are then aggregated over all the portfolio to obtain (3); for a detailed description see ERN (2002). Each one of these component models was validated at every stage of its development by the scientists and engineers who developed them, by comparing model results with actual data from historical events and specific portfolios of property exposures, de Alba and Zúñiga-San Martín (2006).

In very broad terms the simulation algorithm is as follows:

a) Choose an earthquake site at random
b) Given the site, generate a magnitude at random from the corresponding distribution
c) Use the distribution of proportion of losses for the site-magnitude combination to generate a random loss proportion (damage) for each insured building

d) Multiply the proportion resulting in d) by the total value insured for the portfolio and obtain a loss amount.
e) Apply any reinsurance that is in effect.

This process is applied as many times as there are earthquakes in a year to derive a figure of total yearly losses. As many yearly replications are generated as are needed according to the required precision.

Simulation Results

We apply the algorithm to a (disguised) portfolio from a real Mexican insurance company. The portfolio consists of 25,000 buildings. The non-proportional reinsurance scheme (in thousands of dollars) is as shown in Table 1. The insurance company also has a quota share with 10% retention for losses below 7,500, the priority.

Table 1

Layers	Priority	Cover	Reinstatement Premium	RoI	Reins
1	$ 7,500	$ 7,500	$ 1,586	21.15%	2
2	$ 15,000	$ 15,000	$ 1,890	12.60%	2
3	$ 30,000	$ 30,000	$ 2,268	7.56%	1
4	$ 60,000	$ 40,000	$ 1,548	3.87%	1
5	$ 100,000	$ 130,000	$ 2,574	1.98%	1
Superior	$ 230,000	None	NA	NA	NA

The kind of business questions that can be answered with the analysis described in the following lines are of the type: Is total coverage adequate for the company? Does the program have enough reinstatements? How much risk relief is achieved with the program, in monetary terms? How much capital does the company require to guarantee coverage of the insured portfolio?

Applying the algorithm described above and through simple statistical analysis we evaluate the mitigation effect of the reinsurance contract. In Table 2 we show some statistics for the gross losses (without any reinsurance) and for losses net of all reinsurance. In the second row of that table, the latter include reinstatement costs. The third row shows net losses without these costs. The resulting retention level for the whole portfolio is given in the last row.

Table 2

	MEAN	ST. D.	MINIMUM	Q1	MEDIAN	Q3	MAXIMUM
GROSS LOSS	$ 4,873	$ 18,881	$ 6	$ 754	$ 1,565	$ 3,682	$ 1,213,000
NET LOSS	$ 526	$ 5,970	$ 1	$ 75	$ 157	$ 368	$ 762,500
NET LOSS W.O. REINS	$ 516	$ 5,815	$ 1	$ 75	$ 157	$ 368	$ 752,600
RETENTION	10.80%	31.62%	10.00%	10.00%	10.00%	10.00%	62.86%

Table 3 shows gross and net losses for several return periods. This is relevant for complying with the regulatory authority with respect to solvency, that requires a return period of 1500 years in the computation of the PML which in turn is used in calculating the corresponding earthquake catastrophe reserves. Further analysis yields the results in Table 4, where we can see if the reinsurance strategy is what the company needs. One can appreciate that insurance was insufficient in only an extremely low percentage of the total number of years simulated (150,000).

Table 3

Gross Losses	Net Losses	Net Losses without Reinstatement Premiums	% Reduction	Fn	Return Period
$ 300,710	$ 14,278	$ 9,094	95.25%	0.999333333	1500
$ 231,938	$ 10,968	$ 8,448	95.27%	0.999	1000
$ 143,763	$ 9,313	$ 7,827	93.52%	0.998	500
$ 90,040	$ 8,038	$ 7,587	91.07%	0.995	200
$ 78,011	$ 7,684	$ 7,537	90.15%	0.99	100

The table tells us that the relief obtained from the program in this case goes from 90% to 95%, so the program is effective for the company, because with at most $10,000 it can cover a portfolio with a PML of up to $300,710, all in thousands of dollars.

Solvency II states that the Solvency Capital Requirement shall “correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period.” So in this case, the amount should be $8,038 and the relief produced by reinsurance will be $82,002 (90,040 – 8,038), complying with the other Solvency II statement “insurance and reinsurance undertakings shall take account of the effect of risk mitigation techniques, provided that credit risk and other risks arising from the use of such techniques are properly reflected in the Solvency Capital Requirement.” Credit risk adjustments are not illustrated here.

Other regulations apply a higher confidence level, such as the Mexican, which for earthquake is 99.93%. In this case the amount should be $14,278 and the relief produced by reinsurance $286,432.

We also use the Monte Carlo results to show that for a large portfolio and a complicated reinsurance contract, non-proportional reinsurance risk relief can clearly be compared with that achieved with a proportional reinsurance contract, Figure 2[2].

Figure 2

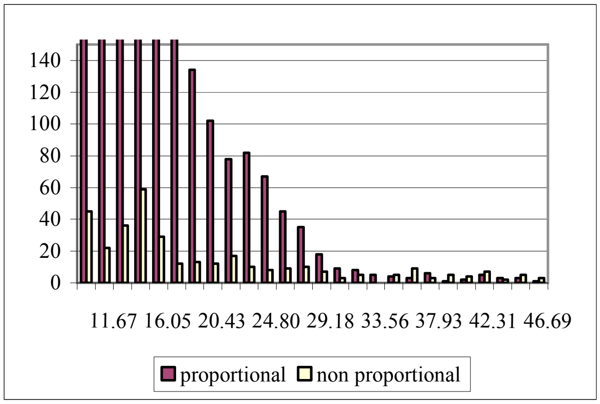

Table 4

Layers	% years all reinstatements were used	% years only one reinstatement was used	% years the second reinstatement was used	Distribution of events by layer
(1)	**(2)**	**(3)**	**(4)**	**(5)**
Priority	NA	NA	NA	99.9937%
1	0.00%	0.49%	0.00%	0.0040%
2	0.18%	0.18%	0.00%	0.0012%
3	0.09%	0.09%	NA	0.0002%
4	0.07%	0.07%	NA	0.0002%
5	0.05%	0.05%	NA	0.0002%
Sup	NA	NA	NA	0.0005%

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[2] Insurer Solvency Assessment Working Party (2004), *A Global Framework for Insurer Solvency Assessment*, International Actuarial Association

As mentioned, Mexican regulation specifies the use of the 0.99933 % quantile to compute the PML and the corresponding reserves. Considering the fact that VaR is an incoherent measure of risk, McNeil et al. (2005), we will now complement the previous analysis using the Tail-VaR, as well as its variance, Manistre and Hancock (2005). In Table 5 we provide the corresponding Tail-VaR for gross and net losses, as well as the variance for the gross losses.

Table 5

Tail-VaR

			Gross Losses				Net Losses			
alpha	Return	k	CTE(alpha)	FSE(CTE)	LL	UL	CTE(alpha)	FSE(CTE)	LL	UL
0.01	100	1500	124,320	3423	117,610	131,030	16,339	1478	13,443	19,236
0.005	200	750	185,873	6051	174,012	197,733	26,399	2909	20,697	32,102
0.002	500	300	309,852	11877	286,573	333,130	53,237	6991	39,535	66,940
0.001	1000	150	442,367	17975	407,136	477,599	96,297	13059	70,703	121,892
0.000667	1500	100	530,843	22049	487,626	574,059	138,066	18178	102,436	173,695

Since for the computations in Table 5, we are using the data from the simulations, the Tail-VaR's are computed as the average of the values exceeding the corresponding quantile. Table 6 shows the results of VaR and tail-VaR for gross and net losses using different return periods.

Table 6

Gross Losses		Net Losses		Return Period
VaR	Tail VaR	VaR	Tail VaR	
$ 300,710	$ 530,843	$ 14,278	$ 138,066	1500
$ 231,938	$ 442,367	$ 10,968	$ 96,297	1000
$ 143,763	$ 309,852	$ 9,313	$ 53,237	500
$ 90,040	$ 185,873	$ 8,038	$ 26,399	200
$ 78,011	$ 124,320	$ 7,684	$ 16,339	100

The results are as expected: the Tail-VaR at each return period is larger than the corresponding VaR. There is no reason to expect the Tail-VaR to be equal to the quantile based PML measure. If these are seen as too large the Tail-VaR will be much more so. It would be necessary to define new criteria for PML calculation if a coherent measure of risk is to be applied. In addition the percentage reduction between the gross and net losses is smaller than when using VaR, but they are still considerably large. In practice, if a quantile based PML is established for a given return period, then a smaller value of the return period might be appropriate for the Tail-VaR.

Although the paper is written in the context of earthquake catastrophe insurance, it is applicable to others, such as hurricane, provided the hurricane model is available. It is shown how a relatively simple simulation model can provide a wealth of information not obtainable by analytic procedures. In fact, the Regulatory Authority has also commissioned a model to evaluate hurricane risk, along the lines of the earthquake model, which is already in use by the market, although without the use of simulations as described above. We have begun to carry out simulation exercises similar to those presented here.

Extreme Value Analysis

The previous analysis provides a large amount of information to the insurer. Yet, there are some technical details that can be explored further. For example, it is know that quantile estimates obtained by simulation are biased, Inui et al. (2005). And the bias tends to zero as the sample size increases. It can also be corrected by bootstrapping, Kim and Hardy (2007). We have used large sample sizes so that the bias should be small. Nevertheless we will carry out additional analyses in order to fine tune the results. In particular the quantile corresponding to the 1500 year return period, which must be used to compute the PML and hence required reserves for earthquake catastrophe risk. We will also complement the analysis with the computation of confidence intervals for the quantiles.

Since PML estimation is essentially an exercise in estimating a large quantile it falls in the field of extreme values. We use the Peaks Over Threshold (POT) method and proceed along the lines set out in Embrechts et al. (1997). We assume that the losses from earthquakes are $X_1, X_2, \ldots, X_n$, i.i.d. with distribution *F(x),* a Generalized Extreme Value Distribution (GEV). Then we choose a high threshold *u*, and so the corresponding excesses are denoted by $Y_i = X_i - u, \quad i = 1, \ldots, n,$; and N_u is the number of exceedances of *u* by $X_1, \ldots, X_n$. The exceedances follow a generalized Pareto distribution (GPD). This GPD, denoted by $G_{\xi,\beta}$, with parameters $\xi \in \mathbb{R}$ and $\beta > 0$ has distribution tail

$$\overline{G}_{\xi,\beta}(x) = \begin{cases} \left(1 + \xi \dfrac{x}{\beta}\right)^{-1/\xi} & \text{if} \quad \xi \neq 0, \\ e^{-x/\beta} & \text{if} \quad \xi = 0, \end{cases} \quad x \in D(\xi, \beta), \qquad (5)$$

where

$$D(\xi, \beta) = \begin{cases} [0, \infty) & \text{if} \quad \xi \geq 0, \\ [0, -\beta/\xi] & \text{if} \quad \xi < 0. \end{cases},$$

Embrechts et al. (1997). We must choose a high threshold *u*, although there are no clear criteria. It should not be too small so as to not produce biased estimators and it should not be too high because it will produce high variance estimators. With our sample of 150,000 we decided to use u = 66.22 (in millions of dollars), which yields N_u = 1000. This value has the added advantage that (empirically) *Prob*{ *X* > 66.22 } = 1/150, and can easily be used to compute our 1500 year return period quantile, as will be shown below. We used the program "ExtRemes" to carry out the analysis, Gilleland and Katz (2005). We then generate the sample of 1000 exceedances that used for fitting the tail of the GPD. Figure 3 shows the histogram, which indicates a good fit.

Figure 3

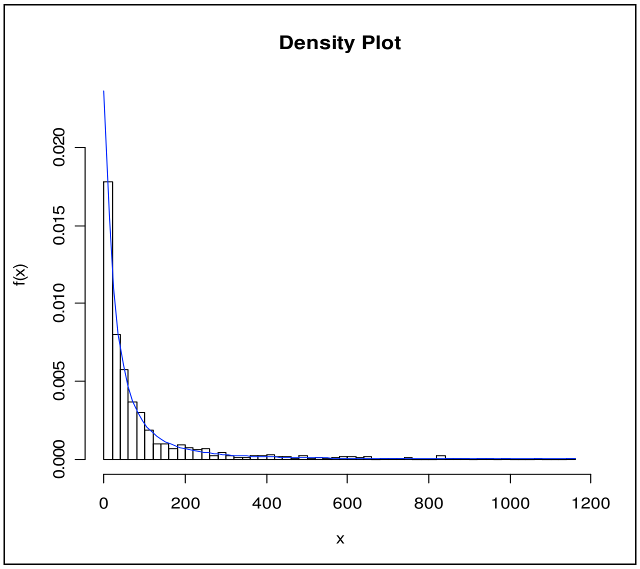


Figure 4 shows the corresponding fit diagnostics. Since we are fitting to the exceedances the two graphs that involve return periods must be viewed with care. In order to compare these results with those obtained by simulation we compute the required quantiles from the GPD. These are given by

$$\hat{x}_p = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}(1-p)\right)^{-\hat{\xi}} - 1\right), \tag{6}$$

Embrechts et al. (1997), where $\hat{\beta}$ *and* $\hat{\xi}$ are the parameter estimates. The value of *p* must be determined such that

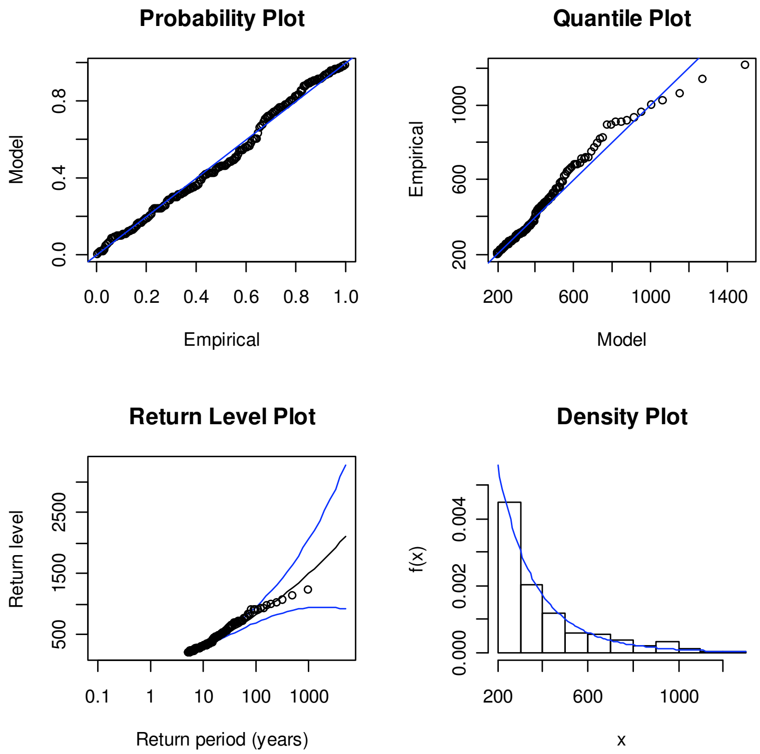
$$\overline{F}(u+y) = \overline{F}(u)\cdot\overline{F}_u(y) = 1-p\ ,$$

where $\overline{F}(x) = 1 - F(x)$, and $\overline{F}(u)$ is estimated by N_u / n = 1/150. For example, to get the quantile for the 1500 year return we take *p* = .999333 and we use an approximation based on (5) to estimate $\overline{F}_u(y)$ and hence the quantile. But from these specifications we end up with

$$0.000667 = 1 - 0.999333 = (1/150)\cdot \overline{F}_u(y)$$

Now we use (6) to obtain a *p'*-quantile for the exceedances (the *Y's*) and where *p'* = 1-0.000667*150 = 1-0.10 = 0.9. This is done with the “extRemes Toolkit”, Gilleland and Katz (2005), by obtaining the 10 year return level. Table 5, shows the return levels, along with their confidence intervals and the simulation results for the return periods 500, 1000 and 1500. These would be the corresponding PML’s for this insurance company using the Mexican earthquake data. Note they are fairly close and in all cases the simulation results are within the 95% confidence intervals.

Figure 4



The result obtained from the ERN program, which is supposed to be exact, turns out to be PML = $ 284,718.06. This is clearly very close to the one obtained using an Extreme Value approach. However the simulation approach yields a wealth of information not obtainable via the ERN program.

Table 7

RETURN PERIODS	SIMULATION	EXTREME VALUE	LOWER LIMIT	UPPER LIMIT	ERN
1500	$ 300,710	$ 280,076	$ 256,524	$ 309,758	$ 284,718
1000	$ 231,938	$ 217,937	$ 203,170	$ 235,519	$ 224,775
500	$ 143,763	$ 141,439	$ 134,747	$ 148,824	$ 144,087

The results in Table 7 show that there is consistency between the different concepts and approaches. The computations obtained via Extreme Value Theory provide additional information to that obtained directly from the simulations and with a formal justification, from the statistical point of view. The simulation results are near the upper bound of the confidence intervals in agreement with the known fact that these have a positive bias. Hence they provide conservative estimates for the PML. The ERN results are roughly in the middle of the interval, but ERN is less flexible for evaluating complex reinsurance schemes.

We have shown how Monte Carlo methods can be used to analyze the effect of complicated reinsurance treaties on a heterogeneous portfolio. Simulation also allows the evaluation of large quantiles although the results may be biased if we do not have a large sample. However this may be further explored via extreme values.

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